FINDING THE EFFECTIVE FRONTIER FOR A PORTFOLIO OF RISK LOANS (ALBANIAN CASE)

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DOI: https://doi.org/10.31410/eraz.2018.137

INTRODUCTION: *The basis for any investment is the desire to benefit from positive returns. This additional return means risk acceptance. So we need to be able to determine both return and risk at the portfolio level. In this section, we portray the portfolio in terms of return and risk and note that risk assessment is a complex problem when the portfolio contains a large number of assets. The reason lies in the fact that the assets are correlated with one another and consequently the correlation will help us to understand the combined returns on the asset portfolio, in other words to understand the risk of the portfolio.*
One of the factors considered when choosing the optimal portfolio for an investor is the level of risk-aversion, the investor's risk preference versus expected return.

Key words: *Optimal Portfolio, Efficient Frontier, Risk.*

LITERATURE REVIEW

The researchers have ascertained the level of non-performing loans as a direct indicator of credit risk. This fact has increased the number of scientific papers related to this area with the purpose of understanding the determining factors of NPL (the level of non-performing loans). Risk management dynamics are the most important factors determining whether a bank will be successful in the market or not. According to the authors Arunkumar and Kotreshwar (2004), the foundation of a proper management lies in the creation of a general working picture defining the institution's priorities, the credit approval process, the credit risk measurement system, and finally a full reporting mechanism. Methods of risk management must evolve and sophisticated with the same steps with which the bank is involved in a new world and risky financial operations.

1. Indifference Curve

The degree of risk aversion is determined by the investor's indifference curve, consisting of the risk / return pair that is defined between return and risk. It shows the growth in return that an investor will seek for an increase in risk level. The optimal portfolio along the efficient frontier is not unique to this model, and depends on the risk / return function of each investor.

Usually the financial theories and the AIMR (Investment and Research Management Association) use the benefit function that yields a portfolio of given expected return $E(r_p)$ and standard deviation σ_p by function of benefit:

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$$U = E(r_p) - 0.005 A\sigma_p^2$$

Where U is the value of the benefit and A is an investor risk aversion. Coefficient 0.005 is agreed with the agreement to allow the expression of expected return and standard deviation as a percentage instead of the decimal numbers. The interpretation is as follows: the benefit of a portfolio increases when the expected return rises and decreases when the variance increases. The relative size of these changes depends on the risk aversion coefficient A. For neutral risk investors, A = 0. Higher levels of risk-aversion are reflected in higher values of A.

Portfolio selection is determined by graphically displaying benefit functions along with the efficient frontier of investment opportunities. In Figure 1, two curves of indifferences named U_1 and U_2 are shown along with the efficient boundary. The curve U_1 has a larger slope, indicating a higher level of risk aversion. The investor is indifferent to any combination of r_p and σ_p over a given curve. The U_2 curve would be appropriate for a no "risk averse" investor, which means that the investor would be willing to accept a relatively higher risk to ensure higher levels of return. The optimum portfolio would be the one that provides the highest benefit - a point in the most northwestern direction (higher return and lower risk). This point will be on the tangent of the benefit curve and the efficient boundary. Each investor logically chooses the optimal portfolio according to his preference for risk-return and none of these portfolios is better than the other is.

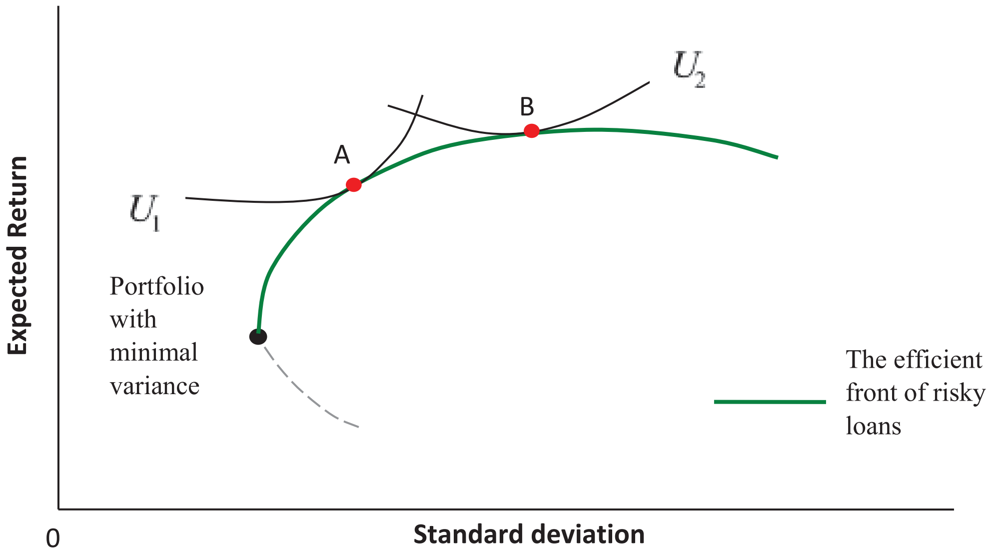

Figure 1.The curbs of indifference and the efficient frontier

2. Optimal Portfolios

To simplify the optimal risk portfolio definition, the capital allocation line (CAL) is used, which presents all possible risk-return combinations available from different asset combination choices to determine the optimal portfolio of risks. Initially, the solving of the portfolio building problem with only two risky assets (in our example, assets A and B) and a risk-free tool will be demonstrated. In this case, an exact formula for the weights of each optimal portfolio asset can be used. This will make it easier to illustrate some general problems related to the portfolio optimization.

The objective is to find the x and y weights that result in the highest slope of the CAL (ie, resulting weights in the riskos portfolio with the highest return-rate variance). Therefore, the objective is to maximize the CALS slope for each potential portfolio, S. Thus, the CALs function:

$$CAL_S = \frac{E(r_p) - r_f}{\sigma_p} \tag{2}$$

For the portfolio of two risky assets, expected return and standard deviation are:

$$E(r_P) = w_A E(r_A) + w_B E(r_B)$$

$$\sigma_P = \sqrt{(w_A{}^2 \sigma_A{}^2 + w_B{}^2 \sigma_B{}^2 + 2 w_A w_B \rho_{AB} \sigma_A \sigma_B)}$$

When maximizing the CAL function, we must meet the limitation that the sum of the weights of the portfolio is equal to one. Consequently, we solve a mathematical problem that is formally written as:

$$\underset{w_i}{Max}\, CAL_S = \frac{E(r_p) - r_f}{\sigma_p}$$

Where $\sum_{i=1}^{2} w_i = 1$. In the case of two risky assets, the solution for the optimal risk portfolio S weights can be shown as follows:

$$w_A = \frac{[E(r_A) - r_f]\sigma_B^2 - [E(r_B) - r_f]\rho_{AB}\sigma_A\sigma_B}{[E(r_A) - r_f]\sigma_B^2 + [E(r_B) - r_f]\sigma_A^2 - [E(r_A) - r_f + E(r_B) - r_f]\rho_{AB}\sigma_A\sigma_B}$$

$$w_B = 1 - w_A$$

Thus, an optimal portfolio is formed from an optimized portfolio of given risk and CAL is generated by a combination of portfolio S and the risk-free tool. Once the optimal S portfolio was built, the aversion level A is used to calculate the optimal portfolio's propensity to invest in the risk component. Suppose that the risk-free rate is r_f, in a portfolio with the expected return rate $E(r_p)$ and the standard deviation σ_p. We will find that for each y choice, the expected return of the portfolio is:

$$E(r_C) = r_f + y[E(r_p) - r_f]$$

The variance for the full portfolio is: $\sigma_C^2 = y^2 \sigma_p^2$.

The investor tends towards maximum benefit, U, choosing the best combination for the risky asset, y. To solve the problem of maximizing utility, more generally, we write the problem like:

$$\max_{y} \text{U}=E(r_C)-0.005A\sigma_C^2$$
$$=r_f+y[E(r_p)-r_f]-0.005Ay^2\sigma_p^2 \; .$$

By extracting the derivative of this expression to zero, we can find y * which gives the optimal position for risk-opposing investors as follows:

$$y^* = \frac{E(r_p)-r_f}{0.01A\sigma_p^2} \qquad (3)$$

The solution indicates that the optimum position on the asset is expected to be non proportional to the level of risk aversion and the level of risk itself (variance), on the other hand, in proportion to the expected return, offered by the risky asset. Once we get to this point, generalization in the case of many risky assets is simple. Before proceeding with it, a summary of the steps that were followed to reach a full portfolio will be presented as follows.

Specify the characteristics of the return of all securities (expected returns, variances, covariances).

We create a risky portfolio.
Calculate the risky optimal portfolio S.
Calculate the characteristics of portfolio S according to the formulas:

$$E(r_P)=w_AE(r_A)+w_BE(r_B)$$

$$E(U(X))=U(m)+\frac{1}{2}U^{//}(m)V(X)+E(R_3(X)) \;\; \text{ku} E(R_3(X))=R_3(X)=\sum_{k=3}^{\infty}\frac{U^{(k)}(m)}{k!}E(R_3(X-m)^k)$$

The combination of funds between the risk portfolio and the risk-free tool.
Calculate the part of the full portfolio, of the combinations within the portfolio S (risk portfolio) and the risk-free tool.
Calculate the distribution of the full portfolio invested in each asset and risk-free tool.

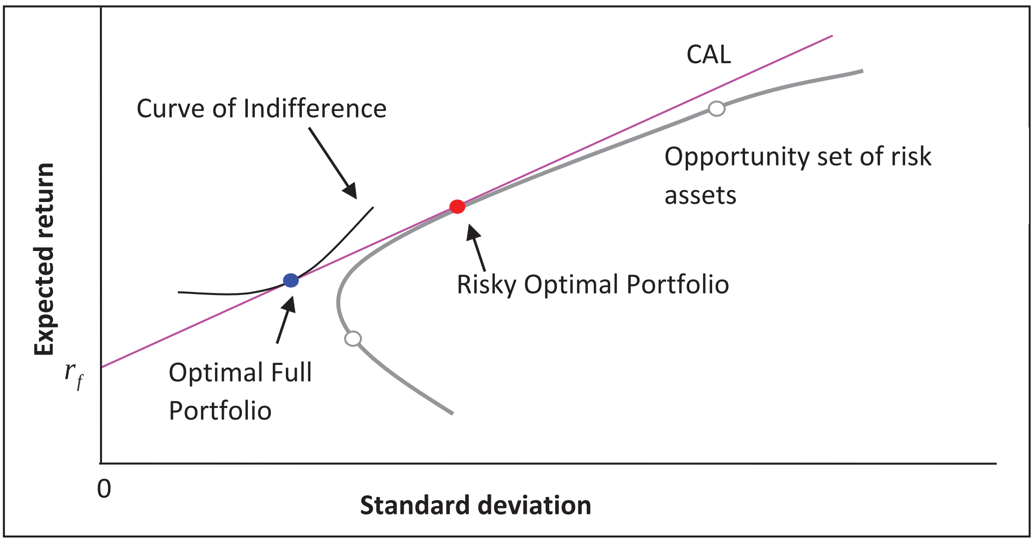

Figure 2: Determining the optimal portfolio

3. Determining Optimal Portfolio (Credins Bank Albania case)

Below will be shown how many efficient multifaceted portfolios can be built.

We have taken into consideration a bank of second level Credins Bank that operates in our market. We have grouped its loan portfolio into five types of loans, grouped by maturity.

- The first group, loans with maturity up to 1 year,
- The second group, loans with a maturity of 1 year to 3 years,
- Third group, loans with a maturity of 3 years up to 5 years,
- The fourth group, loans with a maturity of 5 years to 10 years,
- Fifth Group, loans with a maturity of more than 10 years.

With the monthly data for the respective portfolios from January 2009 to December 2016, we will be able to determine the combination of the Bank's optimum portfolio.

So, solving our problem is to find the optimum portfolio with minimal risk and maximum return, so finding the weights for each grouping in such a way that the bank minimizes credit risk and maximizes its profit.

Step 1: Finding the Efficient Frontier

Initially, we need to calculate the expected return, the standard deviation, and the covariance matrix for historical monthly returns data.

Step 2: Finding the optimal risk portfolio

In order to find the optimal risk portfolio, we need to find the portfolio in the tangent between the CAL and the efficient frontier. For this, Solver can be used. Initially, inserts the target function, ratio reward/ variance ($\frac{E(r_p)-r_f}{\sigma_p}$, assume rate without risk 4.5%), CAL slope.

Step 3: The decision on capital combinations

The decision on capital allocation will be affected by the degree of risk aversion. Now that we have the optimal risk portfolio, we can use the concept of combined funds in the full portfolio between the risk portfolio and the risk-free asset. We use the equation $\boldsymbol{MaxE(X)}$ with condition $\boldsymbol{\phi(X)=\varphi}$ as our function of benefit and we take risk aversion odds to 5 and the risk-free assets rate of 4.5%. Initially we build a complete portfolio with the risk-free tool and with the optimum portfolio of risks.

According to Equation (2), the optimum weight for the risk portfolio is $\frac{E(r_p)-r_f}{0.01A\sigma_p^2}$ and the optimum position of the risk-free asset is 1-$\frac{E(r_p)-r_f}{0.01A\sigma_p^2}$. Then, we use equations:

$$E(r_P)=w_AE(r_A)+w_BE(r_B)$$

$$\sigma_P^2=(w_A\sigma_A+w_B\sigma_B)^2=w_A{}^2\sigma_A{}^2+w_B{}^2\sigma_B{}^2+2w_Aw_B\rho_{AB}\sigma_A\sigma_B$$

to calculate expected return and standard deviation of the full portfolio.

Table 1, 2 and 3 show the average return, risk, correlation matrix, and the covariance matrix for return rates for credit groupings by maturity.

Average return and standard deviation for monthly returns		
Type of the Loan	Average monthly return	Risk
0-1 year	2,54%	8,46%
1-3 years	1,42%	2,51%
3-5 years	0,56%	2,54%
5-10 years	0,51%	2,96%
>10 years	1,16%	2,66%

Table 1. Risk and Average Return on Monthly Returns, Credins Bank Data (Author's Processing)

Correlation matrix for monthly returns of loans on maturity terms					
	0-1 Years	1-3 Years	3-5 Years	5-10 Years	>10 Years
0-1 Years	1,000	0,196	- 0,304	0,099	- 0,034
1-3 Years	0,196	1,000	0,357	0,484	0,323
3-5 Years	- 0,304	0,357	0,010	0,409	0,329
5-10 Years	0,099	0,484	0,409	0,010	0,514
>10 Years	- 0,034	0,323	0,329	0,514	1,000

Table 2: Correlation Matrix, Cedins Bank Data (Author Processing)

Matrix of variance - covariance for monthly returns on loans, by maturity timing.					
	0-1 Years	1-3 Years	3-5 Years	5-10 Years	>10 Years
0-1 Years	0,0071	0,0004	- 0,0006	0,0002	- 0,0001
1-3 Years	0,0004	0,0006	0,0002	0,0004	0,0002
3-5 Years	- 0,0006	0,0002	0,0006	0,0003	0,0002
5-10 Years	0,0002	0,0004	0,0003	0,0009	0,0004
>10 Years	- 0,0001	0,0002	0,0002	0,0004	0,0007

Table 3: Variance-Covariance Matrix, Cedins Bank Data (Author's Processing)

Table 4 gives us the current monthly portfolio with which the bank operates in the credit market, portfolio with minimal variance, portfolio with maximum return and optimal portfolio.

			Weight for each asset					
	Average	Risk	0-1 Years	1-3 Years	3-5 Years	5-10 Years	>10 Years	Profit
Min Variance	1,08%	1,82%	7,35%	23,84%	41,15%	0,00%	27,66%	59,26%
Max Return	2,54%	8,40%	100,00%	0,00%	0,00%	0,00%	0,00%	30,23%
Actual Portfolio	1,23%	4,28%	19,19%	15,20%	16,72%	21,20%	27,69%	53,75%
Optimal Portfolio	1,45%	2,18%	10,67%	55,34%	0,00%	0,00%	33,99%	66,63%

Table 4. Minimum variance portfolio, maximum return and optimal for monthly returns, Credins Bank data (Author's processing)

According to the results of Table 4 we conclude that:

1. If the bank aims to minimize risk, it should distribute its loan portfolio in this way:
    - 7.3% for loans with a maturity of 0-1 year,
    - 23.8% for loans with a maturity of 1-3 years,
    - 41.15% for loans with a maturity of 3-5 years,
    - 27.7% for loans with a maturity of more than 10 years,
    - No loans with maturity of 5-10 years with monthly average return 1.08% and risk 1.82% per month.

    This result is due to the fact that loans with a maturity of 5-10 years are those loans for which the value of the approved principal is high compared to loans with a maturity of less than 5 years and this loan does not necessarily require the mortgage guarantee.

2. If the bank aims to maximize the average monthly return then the portfolio should be allocated 100% for 0-1 year loans. This short-term loans are generally overdrafts and for their approval it is sufficient only for the monthly salary of the borrower. They have the highest return because they also have the highest interest rate of credit and short maturity. Such distribution would bring to the bank a 4.12% increase in monthly risk compared to the current risk of 4.28% of the current portfolio of the bank.

3. If the bank aims to maximize monthly earnings, or as we have called the optimal portfolio, which takes into consideration the return rate for non-risky loans, where in our case we have assumed the average treasury bill rate should be consisting of:
    - 10.7% maturity loans 0-1 year,
    - 55.34% maturity loans 1-3 years,
    - 34% loans with a term of more than 10 years,
    - medium term loans 3-5 years and 5-10 years should be eliminated.

According to this month, the monthly risk shrinks by 2.1% and the monthly profit increases to 13%.

Table 5, 6 and 7 show the average return, risk, correlation matrix and the covariance matrix for the returns of the loan groupings by maturity. Table 8 gives us the current annual portfolio with which the bank operates in the credit market, portfolio with minimum variance, maximum return portfolio and optimal portfolio.

Average Return and Risk for Annual Returns		
Loans	Annual average	Risk
0-1 Years	37,59%	32,23%
1-3 Years	17,04%	20,26%
3-5 Years	7,33%	20,10%
5-10 Years	6,90%	21,33%
>10 Years	13,86%	18,10%

Table 5: Average return and risk for annual returns, Credins Bank data (Author's processing)

Correlation matrix for annual loan returns by maturity term					
	0-1 Years	1-3 Years	3-5 Years	5-10 Years	>10 Years
0-1 Years	1,000	0,172	- 0,185	0,397	0,276
1-3 Years	0,172	1,000	0,720	0,707	0,654
3-5 Years	- 0,185	0,720	1,000	0,656	0,477
5-10 Years	0,397	0,707	0,656	1,000	0,930
>10 Years	0,276	0,654	0,477	0,930	1,000

Table 2: Correlation matrix for annual returns, Credins Bank data (Author's processing)

Matrix of variance - covariance for annual credit returns by maturity term					
	0-1 Years	1-3 Years	3-5 Years	5-10 Years	>10 Years
0-1 Years	0,087	0,009	-0,010	0,023	0,013
1-3 Years	0,009	0,034	0,024	0,025	0,020
3-5 Years	-0,010	0,024	0,034	0,023	0,014
5-10 Years	0,023	0,025	0,023	0,038	0,030
>10 Years	0,013	0,020	0,014	0,030	0,027

Table 7. Matrix of variance - covariance for annual returns, Credins Bank data (Author's processing)

			Weights for each asset					
	Average	Risk	0-1 Years	1-3 Years	3-5 Years	5-10 Years	>10 Years	Profit
Min Varianc e	15,92%	13,50 %	21,67%	0,00%	47,19 %	0,00%	31,15 %	117,94 %
Max Return	37,59%	29,42 %	100,00 %	0,00%	0,00%	0,00%	0,00%	127,79 %
Actual Portfolio	30,25%	24,21 %	19,19%	15,20 %	16,72 %	21,20 %	27,69 %	107,75 %
Optimal Portfolio	25,45%	17,30 %	48,59%	32,01 %	14,76 %	0,00%	4,64%	147,11 %

Table 3. Portfolio with minimal variance, maximum return and optimal return for annual returns, Credins Bank Data (Author's processing)

5. Conclusions and Recommendations

a) According to the results of Table 8 on annual returns on loans, we conclude that:
If the bank wants to minimize risk, it should distribute its loan portfolio in this way:
- 21.67% Assets with a maturity of 0-1 year,
- 47.2% Assets with a maturity of 3-5 years,
- 31.15% Assets with a maturity of more than 10 years,
- Should not hold loans with maturity of 1-3 years and 5-10 years with average monthly return of 15.92% and risk 13.5% per annum.

This distribution reduces the credit risk by 10.7% and the bank's profit by 10% compared to the portfolio available from the bank.

b) If the bank aims to maximize average annual return then the portfolio should be allocated 100% for 0-1 year loans. The same result as well as for monthly returns, therefore maturing loans with less than a year have the highest return by 37.6% but even higher risk level of 29.4%.The bank's profit would be 20% higher than actual.

c) If the bank aims to maximize the annual profit, which takes into consideration the return rate for non-risky loans, where in our case we have assumed the average treasury bill rate should be comprised of:
- 48.6% maturity loans 0-1 year,
- 32% maturity loans 1-3 years,
- 14.8% loans with a term of 3-5 years,
- 4.7% loans with a maturity of more than 10 years,
- loans with maturity of 5-10 years have to be eliminated.

According to this distribution, the annual risk rises to 17.3%, which is approximately 7% less than the current risk, and the annual profit rises to 40%.

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Credins Bank has to invest 23% of its capital in non-risky loans and 77% in riskier loans in order to achieve total portfolio return of 16.89% and risk 14.19% to reach a 12.4% profit level. Compared to the current risk at the level of 24.2% that the bank has on loans this combination of portfolio would reduce the risk by 10%.

Recommendations

Problem loans as a starting point have the moment of receiving the loan. Loan brokers need to be very responsive and skilled in relation to the moment of consideration of applicants' claims. At this moment they have to face two careful analyzes, that of the financial situation of the applicant and the identification of his specific or social characteristics. In assessing the financial aspect of the applicant, the loan agent should be very careful in seeking information on the economic situation and to properly analyze the factors that may affect the future. This is more complicated in business cases, especially in Albania. Currently, many Albanian businesses operate with 2 balances, one for governmental effects and one for internal effects. In the first, they have a pronounced tendency to reduce profits and worsen indicators as a way to evasion. In the second balance they claim to show the real situation and it is they who submit to the banks. The agent should be in a position to carefully analyze the indicators to look at different manipulation with indexes.

6. Literature

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