GOLD PRICE AND THE CHAOTIC GROWTH MODEL

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Abstract: *The basic aims of this paper are: firstly, to create the simple chaotic gold price growth model that is capable of generating stable equilibria, cycles, or chaos; secondly, to analyze the local stability of gold price in the period 2001-2015; and thirdly, to discover the equilibrium gold price with Elliott wave logic in the observed period. This paper confirms the existence of the stable convergent fluctuations of the gold price in the observed period. Also, the golden ratio can be used to define the equilibrium gold price in the presented chaotic model.*

Keywords: *Gold price, Stability, Elliot waves, Chaos.*

1. INTRODUCTION

Gold seems to have offered a protection inflation. There is a high positive correlation between inflation and the gold price. When inflation rises, the value of currency goes down and therefore people tend to hold gold. Rising or higher levels of inflation tends to push gold prices higher.

On the other hand, the gross domestic product (GDP) is the market value of all final goods and services produced within a country in a specific time period. The gold price is significantly driven by the investment demand. Declining real GDP boosts gold prices, because investors decide to buy gold. They want to protect their capital value. It is supposed that: (i) there is a a negative correlation between the real GDP growth and the price of gold; and (ii) there is a negative correlation between the price of gold and the ratio of the GDP to gold price. In this sense, the price of gold signaled the high probability of the recession and / or expansion.

The basic aim of this analysis is to set up a relatively simple chaotic gold price growth model that is capable of generating stable equilibria, cycles, or chaos. It is important to analyze the local stability of the gold price growth in the period 2001-2015 (https://www.macrotrends.net/1333/historical-gold-prices-100-year-chart).

Chaos theory started with Lorenz's (1963) discovery of complex dynamics arising from three nonlinear differential equations leading to turbulence in the weather system. Li and Yorke (1975) discovered that the simple logistic curve can exhibit very complex behavior. Further, May (1976) described chaos in population biology. Chaos theory has been applied in economics by Benhabib and Day (1981,1982), Day (1982, 1983), Grandmont (1985), Goodwin (1990), Medio (1993), Lorenz (1993), Jablanovic (2011, 2012, 2013, 2016), Puu, T. (2003), Zhang W.B. (2006), etc.

2. THE MODEL

The Phillips curve (1958) is a curve that shows the negative short-run trade-off between inflation (Π) and unemployment (u). To illustrate the short-run and long-run relationship between

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inflation and unemployment, Friedman (1968) and Phelps (1967) includes a new variable into the analysis: expected inflation (Π^e). Expected inflation explains how much people expect the overall price level to change, i.e.:

$$u_t = u_n - \alpha\,(\Pi_t - \Pi^e) \tag{1}$$

Further, it is supposed:

$$u_n = \beta\, u_t \tag{2}$$

$$\Pi^e = \gamma\, \Pi_t \tag{3}$$

Where : u_t – unemployment rate ; u_n – natural rate of unemployment; Π_t – actual inflation; Π^e – expected inflation; α – the coefficient which explains how much unemployment responds to unexpected inflation; β – the coefficient which relates unemployment rate and natural rate of unemployment; γ – the coefficient which explains relation between actual and expected inflation;

Further, it is supposed that the growth rate of unemployment at time t should be proportional to $(1 - u_t)$, the fraction of the labor force that is not used up by the unemployment at time t. Assuming that the unemployment is restricted by the labor force, the growth of the unemployment rate should change according the following equation, after introducing a suitable parameter ρ (Jablanovic, 2011).

$$\frac{u_{t+1} - u_t}{u_t} = \rho\left(1 - u_t\right) \tag{4}$$

Where: u_t – unemployment rate; ρ – the coefficient which explains relation between unemployment rate growth rate and the fraction of the labor force that is not used up by the unemployment at time t.

By substitution one derives:

$$\Pi_{t+1} = \left(1-\rho\right)\Pi_t - \left[\frac{\rho\,\alpha\left(\gamma-1\right)}{\left(1-\beta\right)}\right]\Pi_t^{\,2} \tag{5}$$

It is supposed:

$$p_t = \mu\, \Pi_t \tag{6}$$

Where: p_t – the gold price; Π_t - actual inflation. μ -the coefficient which explains relation between the gold price and actual inflation.

By substitution one derives:

$$p_{t+1} = \left(1-\rho\right)p_t - \mu\left[\frac{\rho\,\alpha\left(\gamma-1\right)}{\left(1-\beta\right)}\right]p_t^{\,2} \tag{7}$$

Further, it is assumed that the current value of the gold price is restricted by its maximal value in its time series. This premise requires a modification of the growth law. Now, the gold price growth depends on the actual gold price, p_t, relative to its maximal size in its time series p^m.

We introduce g, as $g = p / p^m$. Thus, g range between 0 and 1. Again we index g by t, i.e. write g_t to refer to the size at time steps t = 0,1,2,3... Now the gold price growth rate is measured as:

$$g_{t+1} = (1-\rho)\, g_t - \mu\left[\frac{\rho\,\alpha(\gamma-1)}{(1-\beta)}\right] g_t^2 \tag{8}$$

This model given by equation (8) is called the logistic model. For most choices of α, β, γ, μ, and ρ, there is no explicit solution for (8). Namely, knowing α, β, γ, μ, ρ, and measuring g_0 would not suffice to predict g_t for any point in time, as was previously possible. This is at the heart of the presence of chaos in deterministic feedback processes. Lorenz (1963) discovered this effect - the lack of predictability in deterministic systems. Sensitive dependence on initial conditions is one of the central ingredients of what is called deterministic chaos.

3. THE LOGISTIC EQUATION

The logistic map is often cited as an example of how complex, chaotic behavior can arise from very simple non-linear dynamical equations. The map was popularized in a seminal 1976 paper by the biologist Robert May. The logistic model was originally introduced as a demographic model by Pierre François Verhulst.

It is possible to show that iteration process for the logistic equation:

$$z_{t+1} = \pi\, z_t\,(1 - z_t),\ \pi \in [\,0,\ 4\,],\ z_t \in [0,\ 1] \tag{9}$$

is equivalent to the iteration of growth model (7) when we use the identification

$$z_t = \mu\left[\frac{\rho\,\alpha(\gamma-1)}{(1-\rho)(1-\beta)}\right] g_t \ \text{ and } \pi = (1\text{-}\rho) \tag{10}$$

Using (8) and (10) we obtain:

$$\begin{aligned} z_{t+1} &= \mu\left[\frac{\rho\,\alpha(\gamma-1)}{(1-\rho)(1-\beta)}\right] g_{t+1} = \\ &= \mu\left[\frac{\rho\,\alpha(\gamma-1)}{(1-\rho)(1-\beta)}\right]\left\{(1-\rho)\, g_t - \mu\left[\frac{\rho\,\alpha(\gamma-1)}{(1-\beta)}\right] g_t^2\right\} = \\ &= \mu\left[\frac{\rho\,\alpha(\gamma-1)}{(1-\beta)}\right] g_t - \left(\frac{\mu^2}{1-\rho}\right)\left[\frac{\rho\,\alpha(\gamma-1)}{(1-\beta)}\right]^2 g_t^2 \end{aligned}$$

On the other hand, using (9) and (10) we obtain:

$$\begin{aligned} z_{t+1} &= \pi\, z_t\,(1 - z_t) = (1-\rho)\mu\left[\frac{\rho\,\alpha(\gamma-1)}{(1-\rho)(1-\beta)}\right] g_t \left\{1 - \mu\left[\frac{\rho\,\alpha(\gamma-1)}{(1-\rho)(1-\beta)}\right] g_t\right\} = \\ &= \mu\left[\frac{\rho\,\alpha(\gamma-1)}{(1-\beta)}\right] g_t - \left(\frac{\mu^2}{1-\rho}\right)\left[\frac{\rho\,\alpha(\gamma-1)}{(1-\beta)}\right]^2 g_t^2 \end{aligned}$$

Thus, we have that iterating (8) is really the same as iterating (9) using (10). It is important because the dynamic properties of the logistic equation (9) have been widely analyzed (Li and Yorke (1975), May (1976)).

It is obtained that: (i) For parameter values $0 < \pi < 1$ all solutions will converge to $z = 0$; (ii) For $1 < \pi < 3,57$ there exist fixed points the number of which depends on μ; (iii) For $1 < \pi < 2$ all solutions monotonically increase to $z = (\pi - 1) / \pi$; (iv) For $2 < \pi < 3$ fluctuations will converge to $z = (\pi - 1) / \pi$; (v) For $3 < \pi < 4$ all solutions will continuously fluctuate; (vi) For $3,57 < \pi < 4$ the solution become „chaotic" which means that there exist totally aperiodic solution or periodic solutions with a very large, complicated period. This means that the path of z_t fluctuates in an apparently random fashion over time, not settling down into any regular pattern whatsoever. Also, for $\pi = 2.6178$ then fluctuations will converge to $z = 0.618$.

The Fibonacci sequence, starting with zero and one, is created by adding the previous two numbers (0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89,144, 233, 377, …). This sequence is significant because of the golden ratio. The ratio of any number in the Fibonacci sequence relative to the number directly to its right is approximately 0.618. The Golden Ratio (golden mean, golden number, golden proportion) is 0.618. Adding the number 1 to the Golden Ratio = *Phi* (Φ =1.618). Both 0.618 and 1.618 are used interchangeably to represent the golden ratio because they represent the same geometric relationship (Lidwell, Holden, and Butler, 2010, p.114).

4. EMPIRICAL EVIDENCE

The main aim of this paper is to analyze the gold price growth stability in the period 2001-2015, by using the presented non-linear, logistic gold price growth model (11):

$$g_{t+1} = \pi\, g_t - \upsilon\, g_t^{\,2} \qquad (11)$$

where: g – the gold price, $\pi = (1- \rho)$, $\upsilon = [\rho\ \alpha\ (\gamma - 1)] / (1 - \beta)$.

Figure 1. Gold Prices - 100 Year Historical Chart of historical data for real (inflation-adjusted) gold prices per ounce back to 1915. The series is deflated using the headline Consumer Price Index (CPI) with the most recent month as the base. The current month is updated on an hourly basis with today's latest value. The current price of gold as of June 12, 2020 is $1,735.85 per ounce.

Source: https://www.macrotrends.net/1333/historical-gold-prices-100-year-chart

Waves can be identified in the gold price movements. The Elliot Wave Theory identifies (Frost A.J. & R.P. Prechter, 2006): (i) impulse waves that set up a pattern; and (ii) corrective waves that oppose the larger trend. The gold price movements are divided into: (i) trends (five waves in the direction of the main trend); and (ii) corrections (three corrective waves) (see Fig.2.).

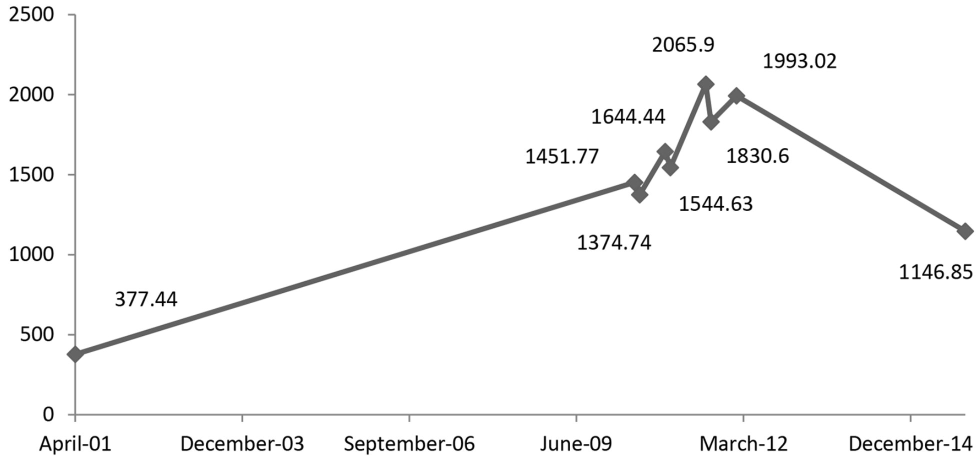

Figure 2. Gold prices 2001-2015.

Source: https://www.macrotrends.net/1333/historical-gold-prices-100-year-chart

Further, data on the gold price per ounce are transformed (Source: https://www.macrotrends.net/1333/historical-gold-prices-100-year-chart) from 0 to 1, according to our supposition that actual gold price, p, is restricted by its highest value in the time-series, p^m. Further, we obtain time-series of $g = p / p^m$ (see Table 1).

Table 1. Data on the gold price per ounce are transformed

	p	g
April,2001	377.44	0.1827
Jun, 2010	1451.77	0.70273
July, 2010	1374.74	0.665444
December, 2010	1644.44	0.795992
January, 2011	1544.63	0.747679
August, 2011	2065.9	1
September, 2011	1830.6	0.886103
February, 2012	1993.02	0.964722
November, 2015	1146.85	0.555133

Source: https://www.macrotrends.net/1333/historical-gold-prices-100-year-chart

Now, the model (11) is estimated (see Table 2.).

Table 2. The estimated model (11)

$N = 8$	π	υ
Estimate	2.518129	1.810815
Std. Err.	0.622875	0.7213
t (6)	4.042755	2.510405
p-level	0.006782	0.045878

The gold price was increased from 377.44 (April, 2001) to 1,146.85 (November, 2015) (unit of measure: **$** per ounce). According to the logistic equation, for $2 < \pi < 3$ fluctuations converge to $z = (\pi - 1) / \pi$, or (2.518129-1)/ 2.518129=0.6029. According to (10), the gold price fluctuations were converged to 0.6029 / (1.810815 /2.518129), or 0.6029 / 0.7191 or 0.8384 or 0.8384 × 2065.9 or **$**1,732.05056 per ounce.

5. CONCLUSION

This paper suggests conclusion for the use of the chaotic gold price growth model in predicting the fluctuations of the gold price. The model (8) has to rely on specified parameters α, β, γ, μ and ρ, and initial value of the gold price, g_0. But even slight deviations from the values of parameters: α, β, γ, μ, and ρ and initial value of the gold price, g_0, show the difficulty of predicting a long-term gold price.

A key hypothesis of this work is based on the idea that the coefficient $\pi = (1 - \rho)$ plays a crucial role in explaining local stability of the gold price, where, ρ is the coefficient which explains relation between unemployment rate growth rate and the fraction of the labor force that is not used up by the unemployment at time t.

The estimated value of the coefficient π is 2.518129. This result confirms continuous fluctuations of the gold price in the period 2001-2015. The equilibrium gold price is $1,732.05056 per ounce in the observed period.

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