8th International Scientific Conference – ERAZ 2022 – Conference Proceedings
DOI: https://doi.org/10.31410/ERAZ.2022.31

A Nonlinear Inflation Growth Model

Vesna Jablanovic[1]

Received: June 27, 2022
Revised: October 7, 2022
Accepted: October 8, 2022
Published: December 30, 2022

Keywords:
Inflation;
Growth;
Chaos

Abstract: *In the short run, the fall in aggregate demand leads to falling output and price level and rising unemployment. In this sense, it is important to use the Phillips curve, i.e., the curve that shows the short-run relationship between inflation and unemployment. This paper creates a relatively simple chaotic inflation rate growth model. Also, this paper examines the inflation rate growth stability in the period 2000-2021 in France, and confirms the existence of the convergent fluctuations of the inflation rate in France in the observed period.*

1. INTRODUCTION

Eser, F., Karadi, P., Lane, P.R., Moretti, L., & C. Osbat (2020) explain the Phillips Curve in the formulation of monetary policy at the ECB. They use the structural Phillips Curve, and identify the slope of the structural Phillips Curve by exploiting cross-country variation and by using monetary policy surprises. Also, they present the role of inflation expectations. They conclude that inflation expectations remain central to successful inflation stabilization.

Del Negro, M., Lenza, M., Primiceri, G.E., and A. Tambalotti. (2020) explain the origins of this disconnect between inflation and economic activity. Namely, the unemployment rate has fallen, but U.S. inflation hasn't responded to this steep drop in joblessness. They conclude that unemployment needs to get lower to bring inflation back to target after a recession. They use an econometric model to explore how monetary policy should adapt.

Coibion, O., Gorodnichenko, Y., & Ulate, M., (2019) show surveys of household or firm expectations for 18 countries to estimate an expectations-augmented Phillips curve. They find strong evidence of a Phillips curve relationship: the Phillips curve is alive and well.

Passamani, G., Sardone, A. & Tamborini, R. (2022) create the Phillips Curve model, in which expected inflation, instead of being treated as an exogenous explanatory variable of actual inflation, is endogenized. This model is tested with the Euro-Zone data 1999–2019 and explains the "inflation puzzles".

Gordon, R.J., (2013) shows that he greatest failure in the history of the PC occurred not within the past five years but rather in the mid-1970s, when the predicted negative relation between inflation and unemployment turned out to be utterly wrong. Instead, inflation exhibited a strong positive correlation with unemployment. The model's simulation success is furthered here by recognizing the greater impact on inflation of short-run unemployment (spells of 26 weeks or

[1] University of Belgrade, Faculty of Agriculture, Nemanjina 6, 11081 Belgrade, Serbia

less) than of long-run unemployment. The implied NAIRU for the total unemployment rate has risen since 2007 from 4.8 to 6.5 percent, raising new challenges for the Fed's ability to carry out its dual mandate

The negative short-run relationship between the rate of inflation and the unemployment rate was developed by A. W. Phillips (1958). Further, Samuelson and Solow (1960) showed a similar inverse correlation between inflation and unemployment in data for the United States. Milton Friedman (1968) argued that monetary policy can pick a short-run combination of inflation and unemployment on the Phillips curve. Edmond Phelps (1957, 1967.1968) also denied the existence of long-run trade-off between inflation and unemployment. Their views imply that a long-run Phillips curve is vertical.

This paper uses logistic mapping as an interesting tool of chaos theory. Chaos theory has been applied in economics by Benhabib and Day (1981,1982), Day (1982, 1983), Grandmont (1985), Goodwin (1990), Medio (1993), Lorenz (1993), Jablanovic (2016), Klioutchnikov, I., Molchanova, O., Klyuchnikov, O. (2017), Puu, T. (2003), Zhang W.B. (2012), etc.

2. THE MODEL

The chaotic inflation growth model is presented as:

$$u_t - u^n = - \alpha \left(\Pi_t - \Pi^e \right) \quad \alpha > 0 \tag{1}$$

$$u^n = \beta\, u_t \quad \beta > 0 \tag{2}$$

$$\Pi^e = \gamma\, \Pi_t \quad \gamma > 0 \tag{3}$$

Π – actual inflation, Π^e – expected inflation, u – the unemployment rate, u_n – the natural rate of unemployment, α is a parameter which shows the responsiveness of unemployment to inflation; β is a parameter which shows the responsiveness of u the natural rate of unemployment to the current unemployment rate; γ is a parameter which shows the responsiveness of the expected rate of inflation to the current rate of inflation.

(1) shows the short-run Phillips curve; α is a parameter that shows the responsiveness of unemployment to inflation; (2) shows the natural rate of unemployment; (3) shows the expected rate of inflation.

According to (1)-(3), it is obtained that:

$$u_t = \Pi_t \left[\frac{\alpha \left(\gamma - 1 \right)}{1 - \beta} \right] \tag{4}$$

Where Π- actual inflation, u – the unemployment rate, α is a parameter that shows the responsiveness of unemployment to inflation; β is a parameter that shows the responsiveness of u the natural rate of unemployment to the current unemployment rate; γ is a parameter which shows the responsiveness of the expected rate of inflation to the current rate of inflation.

Further, it is postulated that the growth rate of the unemployment rate at time t should be proportional to 1 - u_t (the fraction of the labor force that is not used up by the unemployment at time t). After introducing a suitable parameter δ, we obtain:

$$\frac{u_{t+1}-u_t}{u_t} = \delta\,(1 - u_t) \tag{5}$$

Solving the last equation (5) yields the unemployment rate growth model, i.e.,

$$u_{t+1} = (1+\delta)\,u_t - \delta\,u_t^{2} \tag{6}$$

Now, putting (1)-(6) together we immediately get:

$$\Pi_{t+1} = (1+\delta)\Pi_t - \left[\frac{\alpha\,\delta\,(\gamma-1)}{1-\beta}\right]\Pi_t^{\,2} \tag{7}$$

This model given by equation (7) is called the logistic model. Lorenz (1963) discovered the lack of predictability in deterministic systems.

Further, it is assumed that the current value of the inflation rate is restricted by its maximal value in its time series. We introduce π as $\pi = \Pi/\Pi^{m}$. Thus π range between 0 and 1. Again we index π by t, i.e., write π_t to refer to the size at time steps t = 0,1,2,3, ... Now growth rate of the inflation rate is measured as:

$$\pi_{t+1} = (1+\delta)\pi_t - \left[\frac{\alpha\,\delta\,(\gamma-1)}{1-\beta}\right]\pi_t^{\,2} \tag{8}$$

This model given by equation (8) is called the logistic model.

3. THE LOGISTIC MAP

It is possible to show that iteration process for the logistic map:

$$z_{t+1} = \mu\, z_t\,(1 - z_t)\,,\ \mu \in [0,4]\,,\ z_t \in [0,1] \tag{9}$$

is equivalent to the iteration of growth model (8) when we use the identification:

$$z_t = \left[\frac{\alpha\,\delta\,(\gamma-1)}{(1+\delta)(1-\beta)}\right]\pi_t \ \text{ and }\ \mu = (1+\delta) \tag{10}$$

Using (8) and (10) we obtain:

$$z_{t+1} = \left[\frac{\alpha\,\delta\,(\gamma-1)}{(1+\delta)(1-\beta)}\right]\pi_{t+1} = \left[\frac{\alpha\,\delta\,(\gamma-1)}{(1+\delta)(1-\beta)}\right]\left\{(1+\delta)\pi_t - \left[\frac{\alpha\,\delta\,(\gamma-1)}{1-\beta}\right]\pi_t^{\,2}\right\} =$$

$$= \left[\frac{\alpha\,\delta\,(\gamma-1)}{(1-\beta)}\right]\pi_t - \left[\frac{\alpha^2\,\delta^2\,(\gamma-1)^2}{(1+\delta)(1-\beta)^2}\right]\pi_t^{\,2}$$

On the other hand, using (9) and (10) we obtain:

$$z_{t+1} = \mu z_t(1 - z_t) = (1+\delta)\left[\frac{\alpha\,\delta(\gamma-1)}{(1+\delta)(1-\beta)}\right]\pi_t\left\{1-\left[\frac{\alpha\,\delta(\gamma-1)}{(1+\delta)(1-\beta)}\right]\pi_t\right\}=$$

$$=\left[\frac{\alpha\,\delta(\gamma-1)}{(1-\beta)}\right]\pi_t-\left[\frac{\alpha^2\,\delta^2(\gamma-1)^2}{(1+\delta)(1-\beta)^2}\right]\pi_t^{\,2}$$

Thus we have that iterating (8) is really the same as iterating (9) using (10). It is important because the dynamic properties of the logistic equation (9) have been widely analyzed (Li and Yorke (1975), May (1976)).

4. EMPIRICAL EVIDENCE

This paper creates a simple chaotic inflation growth model and confirms the existence of the convergent fluctuations of the inflation rate in France in the observed period. In this sense,

$$\pi_{t+1} = \mu\,\pi_t - \omega\,\pi_t^{\,2}\ \mu, \omega > 0 \tag{11}$$

where π - the inflation rate , μ = (1 + δ) and ω = [α δ (γ – 1)] / (1 – β) ; α is a parameter which shows the responsiveness of unemployment to inflation ; β is a parameter which shows the responsiveness of u the natural rate of unemployment to the current unemployment rate; γ is a parameter which shows the responsiveness of the expected rate of inflation to the current rate of inflation.

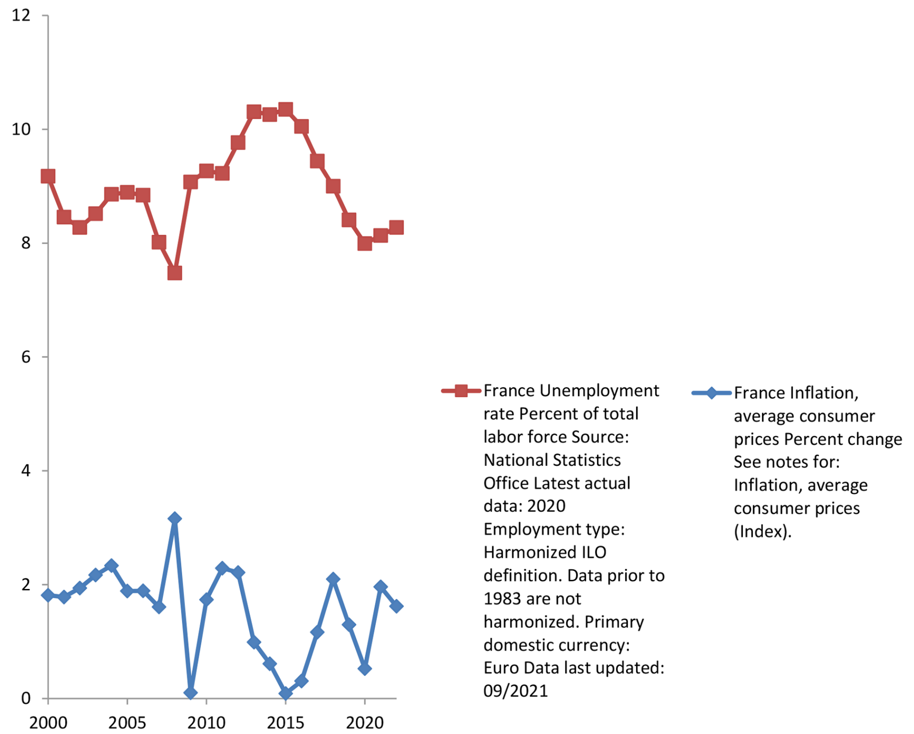

Figure 1. Inflation, average consumer prices, (% change) and unemployment rate (% of total labor force): France. 2000-2021

Source: www.imf.org

Now, we estimate the model (11). The results are presented in table 1.

Table 1. The estimated model (11): Inflation, average consumer prices, % change: France, 2000-2021

R=0.45815		
N=22	π	ω
Estimate	**2.308603**	2.201169
Std.Err.	0.368858	0.531581
t(20)	6.258790	4.140795
p-level	0.00000	0.000506

Because μ= 2.308603, it can be concluded that convergent fluctuations of the inflation rate existed in France in the observed period.

5. CONCLUSION

This paper creates the chaotic inflation growth rate model (8). A key hypothesis of this paper is based on the idea that the coefficient μ = (1+δ) plays a crucial role in explaining local growth stability of the inflation rate, where, δ is a parameter that shows the responsiveness of the unemployment growth rate to the difference between 1 and the unemployment rate, u. The estimated value of the coefficient μ was 2.308603. In his case, the convergent fluctuations of the inflation rate existed in France in the period 2000-2021.

REFERENCES

Benhabib, J., & Day, R. H. (1981). Rational choice and erratic behaviour. *Review of Economic Studies*, 48, 459-471.

Benhabib, J., & Day, R. H. (1982). Characterization of erratic dynamics in the overlapping generation model. *Journal of Economic Dynamics and Control*, 4, 37-55.

Coibion, O., Gorodnichenko, Y., & Ulate, M., (2019.) "Is Inflation Just Around the Corner? The Phillips Curve and Global Inflationary Pressures." AEA Papers and Proceedings, 109: 465-69. DOI: 10.1257/pandp.20191055

Day, R. H. (1982). Irregular growth cycles. *American Economic Review*, 72, 406-414.

Day, R. H. (1983). The emergence of chaos from classical economic growth, *Quarterly Journal of Economics*, 98, 200-213.

Del Negro, M., Lenza, M., Primiceri, G.E., and A. Tambalotti. (2020). "What's up with the Phillips Curve?" Brookings Papers on Economic Activity, Spring, 301-373.

Eser, F., Karadi, P., Lane, P.R., Moretti, L., & C. Osbat (2020), The Phillips Curve at the ECB. Working Paper Series, No 2400 / May 2020, the European Central Bank (ECB).

Friedman, M. (1968). The role of monetary policy. *American Economic Review* 68 (1), 1–17.

Goodwin, R. M. (1990). *Chaotic economic dynamics*. Oxford: Clarendon Press.

Gordon, R.J., (2013) The Phillips Curve is Alive and Well: Inflation and the NAIRU During the Slow Recovery, NBER Working Paper No. 19390 August 2013

Grandmont, J. M. (1985). On enodgenous competitive business cycle. *Econometrica*, 53, 994-1045.

Jablanovic, V. (2016), *A Contribution to the Chaotic Economic Growth Theory*. Roma: Aracne editrice S.r.l.

Klioutchnikov, I., Molchanova, O., Klyuchnikov, O. (2017). Probability of financial stability and security: concepts and models. *Finance and Business,* No. 1, 70-81.

Li, T., & Yorke, J. (1975) Period three implies chaos. *American Mathematical Monthly,* 8, 985-992.

Lorenz, E. N. (1963). Deterministic Nnperiodic Flow, *Journal of Atmospheric Sciences*, 20, 130-141.

Lorenz, H. W. (1993). *Nonlinear dynamical economics and chaotic motion* (2nd ed.). Heidelberg: Springer-Verlag.

May, R. M. (1976). Mathematical models with very complicated dynamics. *Nature*, 261, 459-467.

Medio, A. (1993). *Chaotic dynamics: Theory and applications to economics*. Cambridge: Cambridge University Press.

Passamani, G., Sardone, A. & Tamborini, R. (2022) Inflation puzzles, the Phillips Curve and output expectations: new perspectives from the Euro Zone. Empirica 49, 123–153 (2022). https://doi.org/10.1007/s10663-021-09515-8

Phelps, E. S. (1967), Phillips Curves, Expectations of Inflation and Optimal Unemployment over Time, *Economica,* 34, 254-281.

Phelps, E. S. (1968). Money-wage Dynamics and Labor-market Equilibrium. *Journal of Political Economy* 76: July/August, no. 4, part II, 678-711.

Phillips, A. W. (1958). The Relation between Unemployment and the Rate of Change of Money Wage Rates in the United Kingdom, 1861-1957, *Economica* 25: November, no. 100, 283-99.

Puu, T. (2003). *Attractors, Bifurcations, and Chaos - Nonlinear Phenomena in Economics*. Springer.

Samuelson, P. A.& R. M. Solow (1960). Analytical Aspects of Anti-Inflation Policy. *American Economic Review*, *50*(2): 177–194.

Zhang W.B. (2012). *Discrete Dynamical Systems, Bifurcations and Chaos in Economics*, Elsevier B.V.