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Option Pricing Models: The Evolution of the Black-Scholes-Merton Model

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Abstract: *This paper focuses on the development and impact of the Black-Scholes-Merton (Black-Scholes) model in mathematical finance. It begins with an overview of the Black-Scholes model, including its foundational assumptions, the Black-Scholes equation, and its formula for pricing European options. The paper discusses the model's significant advantages, such as its ability to estimate market volatility and provide a self-replicating hedging strategy. It also addresses its limitations, including assumptions of constant volatility and perfect market conditions, which often do not align with real-world scenarios. Finally, this paper reviews advancements that have refined the model, including adjustments for stochastic volatility, price jumps, and market imperfections.*

1. INTRODUCTION

Trading has always been seen as an opportunistic endeavor, mainly fueled by finances and active traders' investments and most of the trading happens in financial markets. Financial markets, as the name itself suggests, are marketplaces that facilitate the buying and selling of different assets such as bonds, stocks, foreign currencies, and derivatives (Howells & Bain, 2007). These markets facilitate and make possible the exchange of different material and monetary funds between investors, lenders, and borrowers, enabling efficient allocation of resources and capital. These markets are also central to the mathematical theory of finance, which is why much effort especially in the last decades, has been devoted to developing techniques and models to forecast and predict market volatility (Karatzas et al., 1998).

Options are one of the key securities of financial markets and they are defined as the financial derivatives that give buyers the right, but not the obligation, to buy or sell an underlying asset at an agreed-upon price and date They are divided into two main categories, respectively a call option and put option. These two option types, in combination with each other form the basis for a wide range of option strategies designed for hedging, income, or speculation (Ross, 2011). Furthermore, options can be categorized according to when they may be exercised by the buyer or their exercise style. In this paper, only the European Option[2] will be discussed.

As previously mentioned, options hold a very important role in financial markets, yet the concepts surrounding their calculation and pricing have historically been complex and difficult to understand and

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2 A European option is a type of options contract that permits investors to exercise their rights only on the contract's expiration date. The holder has the right to buy or sell the underlying asset at a predetermined price, referred to as the option's strike price (Yoshida, 2003).

predict. It wasn't until 1973 that a breakthrough occurred with the introduction of the options exchange, marking a pivotal moment in the evolution of option pricing theory. That year, economists Fischer Black and Myron Scholes published what would become the dominant theory of how to value and price options, known as the Black-Scholes Model (Black & Scholes, 1973). Robert C. Merton, on the other hand, was the first to publish a paper that expanded the mathematical understanding of the options pricing model and introduced the term "Black–Scholes Options Pricing Model" (Bodie, 2019). The equation and model are named after the first two economics, but Merton is sometimes also credited. From this point on, this paper will only refer to the Black-Scholes-Merton Model as only the Black-Scholes Model. This new model provided a new way of valuing European options showing that an option has a unique price, which is not affected by the risk and return on an underlying asset of that particular option (Ross, 2011).

It was however possible to advance the Black-Scholes Model, and one of the scholars who did so was exactly Robert Merton who improved on the model in many important respects, making it more applicable (Bodie, 2019). Following this, numerous other influential papers and extensions of the model were developed.

2. BLACK-SCHOLES MODEL

The Black-Scholes model is a pricing model that was developed in 1973 and framed the development of most subsequent pricing models (Black & Scholes, 1973). Originally, it was used to price European options and was the first widely adopted mathematical formula for pricing options. It is also often credited with driving the growth of options trading and is considered a major milestone in modern financial pricing. The Black-Scholes Model is used to calculate the fair price of options based on six key variables: volatility, option type, underlying stock price, strike price, time to expiration, and the risk-free interest rate. Rooted in the principle of hedging, the model aims to eliminate risks arising from the volatility of the underlying assets and stock options (Durrett, 2019). The Black–Scholes equation, a partial differential equation in the model, can be used to derive the Black–Scholes formula, which offers a theoretical estimate of the price of European-style options (Black & Scholes, 1973). This formula demonstrates that the option has a unique price regardless of the security's risk and expected return (Black & Scholes, 1973). The model's introduction revolutionized financial markets by allowing for more precise pricing of options and laid the groundwork for the development of future option pricing theories. The formula assumes the underlying stock price follows a geometric Brownian motion with constant volatility. The geometric Brownian motion can be shown as follows:

$$dS = \mu S dt + \sigma S dw \quad (1)$$

where S is the current price of the stock, dS the change in the stock price, μ is the expected rate of return, σ is the volatility and dw is the part follows a Wiener process. The first term of Equation (1) is used to model deterministic trends, while the second is often used to model a set of unpredictable events occurring during the geometric Brownian motion (Durrett, 2019). This stochastic process helps to account for the randomness and uncertainty in stock price movements over time, which is a key feature of financial markets. It also provides a foundation for deriving option pricing models by incorporating both expected returns and market volatility.

3. BLACK-SCHOLES ASSUMPTIONS

A derivative from the geometric motion base of the Black-Scholes formula is needed in order to come up with a closed form pricing solution for European call and put options. Before getting into the mathematical details and calculations, it is important to note that the Black-Scholes formula can only be derived and subsequently used to deliver possible results only under specific assumptions. These assumptions that need to be met include: (Del Giudice et al., 2015):

1. Markets are always efficient and open.
2. There are no taxes or transaction costs.
3. The risk-free interest rate is known and remains constant over time.
4. The volatility of the underlying asset's price is known and remains constant.
5. The price movements of the underlying asset follow a lognormal distribution, meaning that the capitalized returns are normally distributed.
6. The option can only be exercised at the expiration date, as it is a European option.
7. There are no dividends during the life of the option (Del Giudice et al., 2015).

4. DEVIATION OF THE BLACK-SCHOLES EQUATION

First note the following variables: S = stock price, t = time, V = V (S, t) = is the price of the option as a function of stock price S and time t, r is the risk-free interest rate, and σ is the volatility of the stock (Hull & Basu, 2016).

Given the complexity and mathematical knowledge required to derive the Black-Scholes Formula through the Black-Scholes equation, this paper will adopt a step-by-step approach to its explanation. By breaking down the technical aspects, the goal is to make the concept accessible and understandable to a broader audience.

The starting point is the Itô's Lemma for two variables (Hull & Basu, 2016):

$$dV = \left(\frac{\partial V}{\partial t} + \mu S\frac{\partial V}{\partial S} + \frac{1}{2}\sigma^2 S^2\frac{\partial^2 V}{\partial S^2}\right)dt + \sigma S\frac{\partial V}{\partial S}\,dw. \tag{2}$$

Now consider a specific portfolio known as the delta-hedge portfolio, which involves being short one option and holding ∂V/∂S shares at time t (Hull & White, 2017). The value of these holdings is:

$$\Pi = -\mathrm{V} + \frac{\partial V}{\partial S}S \tag{3}$$

Over the time period [t, t+Δt], the total profit (or loss) from changes in the values of the holdings is:

$$\Delta\Pi = -\Delta\mathrm{V} + \frac{\partial V}{\partial S}\Delta S \tag{4}$$

In this step it is important to discretize the equations for *dS*/*S* and *dV* by replacing differentials with deltas:

$$\Delta\mathrm{S} = \mu S\Delta\mathrm{t} + \sigma S\Delta\mathrm{w} \tag{5}$$

$$\Delta\mathrm{V} = \left(\mu S\frac{\partial V}{\partial S} + \frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2\frac{\partial^2 V}{\partial S^2}\right)\Delta\mathrm{t} + \sigma S\frac{\partial V}{\partial S}\Delta\mathrm{w} \tag{6}$$

while appropriately substituting them into the expression for ΔΠ:

$$\Delta\Pi = \left(-\frac{\partial V}{\partial t} - \frac{1}{2}\sigma^2 S^2\frac{\partial^2 V}{\partial S^2}\right)\Delta\mathrm{t} \tag{7}$$

Note that the term Δw has disappeared, indicating that uncertainty has been removed and the portfolio is now effectively risk-free (Hull & Basu, 2016). The rate of return on this portfolio must match the rate of return on any other risk-free asset; otherwise, arbitrage opportunities would arise. Assuming a risk-free rate of return r, the following must hold over the time period [t, t+t]:

$$\Delta\Pi = r\Pi\Delta t \tag{8}$$

When substituting the formulas for ΔΠ and Π = ∫ΔΠ, then:

$$\left(-\frac{\partial V}{\partial t} - \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2}\right)\Delta t = r\left(S\frac{\partial V}{\partial S} - V\right)\Delta t. \tag{9}$$

By simplifying the previous equation (9), the renowned Black-Scholes partial differential equation is obtained:

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0. \tag{10}$$

The fundamental financial point of the equation is that by buying and selling in a strategic manner the underlying asset, one can perfectly hedge the option, meaning that one can potentially completely "eliminate risk" (Hull & Basu, 2016). It will be seen later while discussing the limitations of the model in Section 6, that eliminating risk is not feasible in the real-world context.

5. BLACK-SCHOLES FORMULA

As mentioned earlier, the Black–Scholes formula is used to calculate the price of European put and call options, in accordance with the Black–Scholes equation discussed in the previous chapter. In the Black-Scholes model, the value of a call option can be expressed as a function of five variables.

The value of a call option on a non-dividend-paying underlying stock, based on the Black-Scholes parameters, is given as (Black & Scholes, 1973):

$$C(S_t, t) = N(d_1)S_t - N(d_2)Ke^{-r(T-t)} \quad \text{for} \tag{11}$$

$$d_1 = \frac{1}{\sigma\sqrt{T-t}}\left[\ln\left(\frac{S_t}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)\right] \quad \text{and} \tag{12}$$

$$d_2 = d_1 - \sigma\sqrt{T-t} \tag{13}$$

where N is the cumulative distribution of the standard normal distribution, T-t is the time to maturity (expressed in years), S_t is the spot price of the underlying asset, K is the strike price, r is the risk free and σ is the volatility of returns of the underlying asset (Black & Scholes, 1973).

The price of the corresponding put option based on put-call parity is then given by (Jiang, 2019):

$$P(S_t, t) = Ke^{-r(T-t)} - S_t + C(S_t, t) = N(-d_2)Ke^{-r(T-t)} - N(-d_1)S_t \tag{14}$$

6. ADVANTAGES AND LIMITATIONS OF THE BLACK-SCHOLES MODEL

The Black Scholes model is one of the concepts in financial theory that can be said to be a major breakthrough in both the way it is conceived and the way it is applied. Its most important advantage is that it can determine the market volatility of an underlying asset —typically based on price and time—without regard to the investor's attributes such as the expected return, risk attitude, or utility functions (Teneng, 2011).

In this section of the paper, some of the main advantages of the Black Scholes model will be listed, as well as some of its limitations. The key advantages of the model are (Teneng, 2011):

1. Market Volatility Assessment: The Black-Scholes model is able to estimate the market volatility for an underlying asset, typically as a function of price and time, without requiring any identifying characteristics of the investor such as the expected return, risk tolerance, or the utility function.
2. Self-Replicating Strategy (Hedging): The model offers a self-replicating strategy, which allows investors to continuously buy and sell derivatives and underlying assets in a way that replicates the payoff of a derivative security at maturity. This strategy acts as a form of insurance, ensuring that any loss on one side of the portfolio is exactly compensated by a gain on the other side.
3. Absence of Arbitrage Opportunities: The implications of the Black-Scholes model include the absence of arbitrage, which is the ability to make riskless profit in the financial markets – due to the assumption that money movements occur in such a manner that there is a net zero profit or loss to any one portfolio which means that no "risk free" cash is available in the marketplace. Furthermore, the portfolio is self-financing, with no need for additional funds.
4. Wide Usage: The Black-Scholes model has a universal scope as it does not depend on any particular investor classes. This is the reason why it is among the most successful models in finance with a wide range of applications in various markets and instruments.

Despite its widespread use, the Black-Scholes model is based on several unrealistic market assumptions (Teneng, 2011) leading to some model limitations (Janková, 2018):

1. Constant Volatility: The model assumes volatility remains constant over time, which is not the case in reality. Volatility often clusters and is influenced and determined by factors such as trading volume and asset price returns.
2. Random Walk and Independence: It presumes that stock prices follow a random walk and that future prices are independent of past prices. On the contrary, movements in the stock or bond markets respond to macroeconomic events and prices are often auto correlated and are volatile clusters.
3. Log-Normal Distribution: The model assumes returns are log-normally distributed, but real financial data and historical datasets often show heavier tails and finite variance, differing from the log-normal assumption.
4. Constant Interest Rates: It assumes a risk-free rate that is constant, this is not realistic because in the real world, white interest rates, rates do not stay constant.
5. No Dividends: The basic model assumes that the underlying stock does not pay dividends. In practice, more often than not, dividends are a common occurrence, though adjustments can be made to account for them.
6. No Transaction Costs: It also does not account for transaction costs and commissions when engaging in buying and selling which is unrealistic in a real environment.
7. European-Style Options: The model applies to European-style options, which can only be exercised at expiration. It does not account for the flexibility of American-style options that can be exercised at any time.
8. Perfect Liquidity: It assumes markets are perfectly liquid with no constraints on buying or selling amounts, which is not always the case in real markets.

These were some of the advantages and limitations of the Black-Scholes Model discussed in this paper. Inspired by these limitations, many researchers have developed extensions of this model which will be mentioned in the upcoming section.

7. UPGRADES OF THE BLACK-SCHOLES MODEL

Numerous models have sought to address the limitations of the Black-Scholes model, particularly in response to real-world issues such as stochastic volatility, market illiquidity, and market imperfections. The Heston Model, developed in 1993, is one of the notable improvements over the previous model because it introduces stochastic volatility, allowing volatility to fluctuate according to its own stochastic process (Heston, 1993). This model better captures the volatility clustering observed in financial markets compared to the constant volatility assumption of the Black-Scholes model. The Heston model is also less restrictive since it allows for market friction by modelling the correlation between the prices of assets and stochastic volatility (Wilmott, 2000). As a result, it is better suited for capturing the dynamics of volatile markets, accommodating changing volatility levels, and capturing the volatility smile.

Another important extension is the GARCH (Generalized Autoregressive Conditional Heteroskedasticity) model, which models volatility as a function of past returns and past volatility, accommodating time-varying volatility (Duan, 1995).

In addition to stochastic volatility, the Jump-Diffusion Model, introduced by Merton in 1976, addresses the issue of sudden, large price movements by incorporating jumps in asset prices alongside continuous changes (Merton, 1976). This model is one of the first beyond Black-Scholes model in the sense that it tries to capture the negative skewness and excess kurtosis of the log stock price density by a simple addition of a compound Possion jump process (Matsuda, 2004). The Local Volatility Model, developed by Dupire in 1994, allows volatility to vary as a function of both the underlying asset price and time, providing a more accurate fit to market data and addressing discrepancies between implied volatility and observed prices (Dupire, 1994).

In order to consider the issues related to liquidity and market imperfections, models such as the Extended Vasicek Model have taken interest rate dynamics into account while considering liquidity constraints and market frictions (Vasicek, 1977). Vasicek's model was the first to incorporate mean reversion[3], a feature that set interest rates apart from other financial prices. Unlike stock prices, which can potentially increase without limit, interest rates are constrained because excessively high rates would hinder economic activity, leading to a subsequent decline. Similarly, interest rates rarely fall below zero (Vasicek, 1977). Consequently, interest rates tend to fluctuate within a confined range, consistently gravitating back toward a long-term average (Brigo & Mercurio, 2006). These advancements represent significant progress in option pricing theory, offering more realistic frameworks that better reflect the complexities observed in financial markets in the real world.

8. CONCLUSION

To summarize, it can be said that the development of the Black-Scholes model has contributed to the advancement of financial theory in a notable way, providing a practical solution for European option pricing. Nonetheless, its limitations make it hard to apply the model in practice as perfect market conditions or constant volatility and interest rates are rare in real-life financial markets. It

[3] Mean reversion is a financial theory which suggests that, after an extreme price move, asset prices tend to return back to normal or average levels (Exley et al., 2004).

is these limitations that have resulted in more sophisticated models such as stochastic volatility models, jump-diffusion models, and local volatility models which solve the problem of price jumps in an illiquid market. All of these improvements in pricing models for financial options testify to the changing face of financial markets, as well the desire to capture more of the market as it is. As much as the Black-Scholes model has acted as the go-to option valuation model and remains a foundational tool, the subsequent refinements and alternative models that have been developed have gone further in bringing real environments to theories of option pricing concerning the ever-volatile market forces.

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